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Sinusoidal and Pulse Response of a Plate Distillation Column by Reflux Upset

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A study was made to evaluate pulses as a forcing function on a 24-plate distillation column. Pulses of two shapes, rectangular and displaced cosines, and of different widths were used as inputs to the reflux return line from the condenser to the column. The effect of the disturbance was a change in the liquid return rate which correspond to the shape and size of the pulse. The output responses of the system were temperatures measured at different times and at different plates in the column.

Bode diagrams were plotted from the experimentally determined data. From these plots, it was determined that the system could be approximated by linear first-order equations. The time constants for the linear system were determined both by direct sinusoidal forcing and by pulse forcing. Pulse data were considered acceptable when the values of the time constant and the phase angle determined by the pulse compared favorably with those determined by steady state sinusoidal forcing.

The application of pulse inputs to determine the dynamic response of a chemical process system is a known technique in the chemical and petroleum industries. From a theoretical standpoint, the technique makes it possible to define the frequency response over the entire frequency range for a system with the use of a single pulse. This has been demonstrated by Dreifke (*1*), and Clements and Schnelle (*2*) through the analytical analysis of various mathematical models which they subjected to different pulses readily determined. From such studies one can see some of the advantages as well as disadvantages of pulse techniques relative to other methods of analysis such as direct frequency response by sinusoidal forcing or transient response.

When dealing with a complex chemical process or chemical operation, one is confronted with two problems which do not exist with theoretical mathematical models. One, the complete model of the system is rarely known. Further, whereas a pulse of a specified size or shape may not change the model of a theoretically defined system, it may change that of a real system. As an example, the model of a distillation column has been found to be linear (or pseudolinear) at steady state, yet, if disturbed by a large upset, it may become nonlinear during a transient

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period. Also, the system's natural noise usually exceeds that of a computer, or that programmed into a computer, and this causes inaccurate results. A researcher is confronted not only with the complexities of the process or system but also with the limitation of analytical and control equipment which are a part of the system. Yet it is necessary to define the model of such a real system in order to effect efficient and stable controls and, ultimately, optimization. Since pulse inputs are so convenient to establish the model of a system, it is desirable to determine their effectiveness and limitations in studying real systems. It is the objective of this paper to define acceptable criteria for rectangular and displaced cosine pulses in their use on plate distillation columns.

BACKGROUND

The application of pulse technqiues to mechanical, electrical, and regulatory systems has been reported by many investigators (*3* to *9*). Lees and Hougen (*10*) recognized the value of pulse techniques and used pulses to determine frequency response for chemical processes including heat exchangers. Further studies on heat exchangers were conducted by Morris (*11*), and Hougen and Walsh (*12*) who extended the techniques to experimental systems. Two analytical investigations, Clements and Schnelle (*2*), and Dreifke (*1*) have explored in detail the application of a wide range of pulses to various mathematical models simulated on a computer. These studies have demonstrated theoretically the value of pulse techniques and also have provided guidelines on their limitations. Draper, McKay, and Lees (*13*) present an extensive set of reference material on this subject as well as discussion of theoretical concepts. Marino and Stutzman (*14*), present some conclusions on an experimental study performed with rectangular pulses on a plate distillation column.

In the absence of an exact mathematical model to verify pulse data, one can use as a comparison information obtained by direct frequency response tests. Theoretically, there is an optimum pulse shape and width for any system. However, equipment and operation limitations may not allow generation of the desired shape and width so that conventionally generated functions, such as rectangular and displaced cosines, are more likely to be used for input disturbances. The only requirement necessary for pulse testing is that sufficient energy be put into the system to generate measurable outputs. These pulse functions can be handled through an extension of the Fourier or Laplace transforms. If the Fourier transform is used, the integral may be defined as

$$\int_{-\infty}^{\infty} f(t)\,dt$$

and its transform is then defined as

$$FT[f(t)] = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt = F(j\omega)$$

Over a finite time range, this may be expressed as

$$FT[f(t)] = \int_{0}^{T_p} f(t)e^{-j\omega t}dt = F(j\omega)$$

where T_p is the period of the nonzero portion of the pulse. If the integrals of both output, $y(t)$, and input, $x(t)$, exist, and also, if each converge over the time elements considered, T_o and T_p, respectively, the amplitude ratio may be expressed as:

$$\text{A.R.} = |G(j\omega)| = \frac{\sqrt{\left[\int_0^{T_o} y(t)\cos(\omega t)\,dt\right]^2 + \left[\int_0^{T_o} y(t)\sin(\omega t)\,dt\right]^2}}{\sqrt{\left[\int_0^{T_p} x(t)\cos(\omega t)\,dt\right]^2 + \left[\int_0^{T_p} x(t)\sin(\omega t)\,dt\right]^2}}$$

If the mathematical equations of the input and output pulses are known, the integrals may be evaluated exactly. Although the input is known, the output transform must usually be determined from experimental data. Of the several methods available to evaluate the transform numerically, a variation of the TAFT method by Dreifke (*1*) was used in this study and was computer programmed for the IBM 7040.

These experimentally determined data, when presented as Bode diagrams, allows one to extract information about system behavior as well as control loop criteria.

Equipment and Arrangement

The dynamic studies, which included both sinusoidal and and pulse tests, were conducted on a 24-plate, 8 in. diameter distillation column (Figure 1) which was operated at total reflux. Each plate contained five uniformly spaced bubble caps and the plate spacing was 6½ in. A 40 gal. capacity reboiler provided the vapor feed for the column.

Column temperature profiles were recorded on a 24-point Bristol recorder. Temperatures were measured at twelve positions by means of iron-constantan thermocouples placed in the liquid on every other plate in the column. Other recorded temperature points were located in the reboiler, overhead vapor line, inlet reflux line, and the condenser inlet and outlet water lines.

The input and output signals were recorded continuously on a Sanborn 858 multichannel recorder. Output temperature thermocouples were located in the liquid phase on plates 4, 12, and 20 numbered from the top of the column.

Input signals were generated by means of a Hewlett-Packard 202A low frequency function generator. This unit is capable of producing either a continuous sinusoidal output or single pulse signals which are rectangular, displaced cosine or triangular in in shape.

The input pulses and continuous sine waves were introduced into the overhead reflux return line through a Taylor Lin-E-Aire control valve which controlled the reflux flow rate to the column. The electrical signal from the generator was converted to a pneumatic signal at the valve by means of a Leeds and Northrup electropneumatic converter. The variation of valve position was measured by a Sanborn 7DCDT transducer. Although the electrical output signal from the transducer was used in the computations as the input pulse to the system, the linear relation between stem position and flow rate was known.

The material distilled in the column was a mixture of benzene-acetone. This chemical system was selected because benzene and acetone have a wide boiling point temperature spread

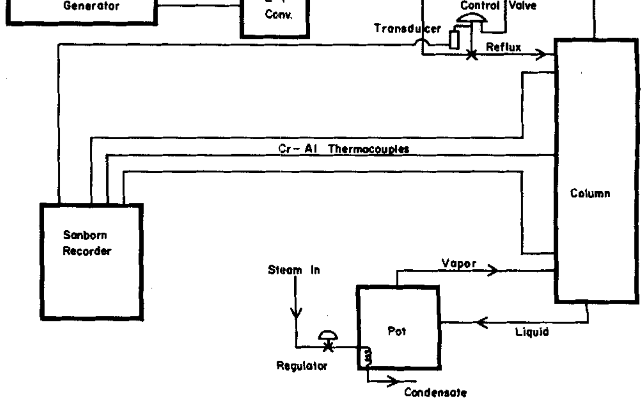

Fig. 1. Plate distillation column and system.

of 43°F. and the equilibrium relationship is nearly linear over the composition range.

The input variations, including both pulse and continuous sinusoidal forcing, were changes in reflux flow rate caused by opening and closing the control valve according to a predetermined time-position relationship. The system output signals were voltages corresponding to temperature changes, as measured by chromel-alumel thermocouples. The time constant, τ_T, for the thermocouple was 0.5 sec. The steam pressure was controlled at 2.2 lb./sq.in. gauge by a Mason-Neilan type 33-1 regulator. Prior to the introduction of flow upsets, steady state was established. This was determined both by constant temperatures as observed on recorders and by chromatographic analysis of liquid composition.

TABLE 1. TIME CONSTANTS AND PHASE ANGLES FOR EXPERIMENTAL RUNS

Runs	τ_1 (2 sec.)	τ_2 B(5 sec.)	Input pulse width (sec.)	Input pulse width/τ	Max. output amp. band width of noise	ϕ (Phase angle) Cal. from the break point of the Bode diagram
Rectangular pulses						
A-1	—	17	70	4.0	21/1	—43°
A-2	23	21	26	1.2	21/1	—50°
A-3	16	18	38	2.0	23/1	—42°
A-4	13	17	84	4.7	33/1	—36°
A-5	—	—	20	—	12/1	—
Displaced cosine pulse						
B-1	25	—	100	4.0	9/1	—48°
B-2	—	—	96	—	5/1	—33°
B-3	—	—	111	—	10/1	—30°
B-4	23	—	125	5.0	10/1	—50°
Direct frequency response						
		τ = 17 sec.		ϕ break point = —50°		

EXPERIMENTAL RESULTS

The classic approach to determing system dynamics in the absence of an exact model is by direct sinusoidal forcing. In order to establish criteria for the use and comparison of pulse data, it was necessary to obtain information on the distillation column behavior. This was done by direct frequency response tests. A range of six frequencies with a fixed amplitude generated the desired data necessary for constructing a Bode diagram. Examination of the sinusoidal data showed that the output frequency was the same as the input frequency and outputs are sinusoidal in shape.

The direct sinusoidal data of Figures 2 and 3 show a slope of —1 and a break point frequency to be located at approximately 50°. Thus, the system can be represented by a linear (or pseudolinear) first-order equation (15).

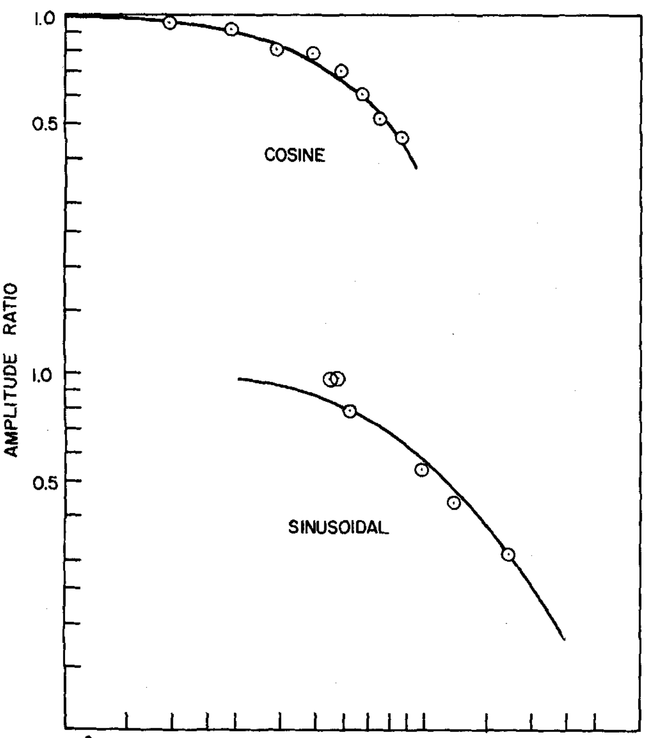

Fig. 2. Bode diagrams. Lower half: direct sinsuoidal forcing. Upper half: displaced cosine pulse, run B-13, Table 1.

A total of thirteen experimental pulse runs were made on the distillation column. Nine are reported in Table 1. These represent five runs, (coded as A) in which rectangular pulses were used and four (coded as B) in which displaced cosines were used. Column headings identify the computed time constants (calculation by the TAFT routine for both 2 and 5 sec. increments of the output response), input pulse width, the ratio of output pulse width to time constant, the ratio of maximum output amplitude to band with of noise, and calculated phase angle. For all of these reported runs, the output signals were measured on the twelfth plate. These were the experiments in which the error in the response (in this case, noise in relationship to output amplitude) was sufficiently small to permit calculation of frequency responses and the construction of Bode diagrams. For all runs, output temperatures were also measured on the fourth and twentieth plates. However, for these plates the amplitude of the response was small relative to the magnitude of error, so that the temperatures measured were inaccurate.

In four of the thirteen experimental runs, attempts were made to force the column with triangular pulses. Although

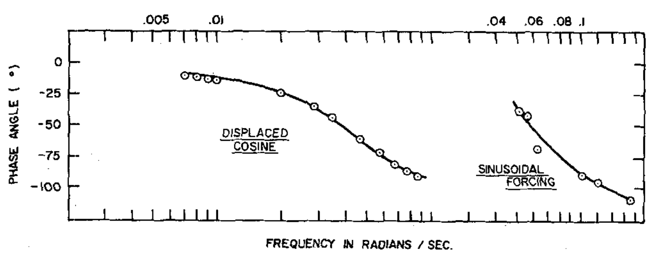

Fig. 3. Phase angle versus frequency plot. Right side section: direct sinusoidal forcing. Left side section: displaced cosine pulse, run B-1, Table 1.

the pulse generator was capable of producing triangularly shaped pulses, the mechanical action of the valve did not permit it to follow this pulse shape. In fact, these pulses were similar to displaced cosines. The results of these runs are not reported in this study because the results agreed with the same criteria as those for the displaced cosines.

The time constants, τ, for the process exclusive of time lags were determined from the break point of the Bode diagrams (see Figure 2 as an example). The time lags were observed directly from the experimental data. Phase angle data for the pulse runs were obtained from the computer routine. The direct frequency response results were used as the reference standards. Acceptable values obtained from pulses are considered to be those which are within 25% of those obtained from direct frequency response.

It is observed that for rectangular input pulses, time constants and phase angles fall within acceptable limits for T_p/τ ratios which do not exceed a value of 5/1, and for output amplitudes/noise ratio of 20/1 or higher. For displaced cosine pulses, the T_p/τ ratio can be somewhat larger, 4/1, and the output amplitude/noise ratio of 9/1.

It is expected that the pulse width for a displaced cosine could be larger than that for a rectangular pulse. Frequency content analysis indicates that the time period to the first zero is larger for displaced cosine than for rectangles. This is one of the reasons for the 70% to 80% of first zero rule proposed by Hougen and Walsh (*12*). Although the output amplitude/noise ratio is smaller for the displaced cosine than for the rectangular pulses, it should be pointed out that for best results, this ratio should be as large as possible. Unfortunately, with limitations imposed by signal generators, transducers, analytical and recording equipment, it is difficult to obtain high ratios for this value when using displaced cosine pulses.

COMPUTER DATA REDUCTION

For the pulse tests, input and output time histories were obtained as a continuous strip chart record. Numeral evaluations of the Fourier transform were performed by reading incremental values of the pulse height and time and transferring these data onto punched cards for evaluation by the TAFT routine.

In order to check the computer routine, tests were conducted with a known mathematical function, and the computer outputs were compared with the theoretical values (*13*).

For a rectangular pulse, the evaluation of the transform is given as $2 \sin \omega T_p / \omega T_p$ where T_p is the width of the pulse. A tabulation of all desired frequencies up to the first zero gave accuracies within 0.1%. Recording intervals of 2 and 5 sec. were used (see columns labeled τ_1 and τ_2 of Table 1) for the calculation of the Fourier transform. These different time intervals did not have any significant effect upon the results. However, in analyzing the output data, judgment must be used so that intervals are small enough to include the irregularities in the response curves, but large enough so that the computer time is reasonable.

CONCLUSION

It was established by this study that the dynamics of a plate distillation column can be determined by experimental pulse techniques. As has been previously reported, it was found experimentally that plate distillation columns are linear (or pseudolinear) over narrow ranges of disturbances, and as a result, Laplace and Fourier transforms may be utilized. These in turn may be used to calculate frequency response and transfer functions. Rectangular and displaced cosine pulses were examined in this study although any pulse shape can be used. For the use of rectangular and displaced cosine pulses, the criteria presented in the section entitled *Experimental Results* are applicable as guidelines to an experimental study.

To experimentally investigate a plate distillation column, the procedure would require the pulsing of the column. From such studies, the time constant would be determined. Pulses of various widths would be used and later compared with the column's time constant to assure that the ratio T_p/τ is met. The maximum amplitude of the response to band width of noise would be calculated as experimental data are observed.

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NOTATION

$f(t)$ = function of time, original dependent variable
t = time, sec.
T_p = width of pulse, sec.
T_o = time in second of output response
j = $\sqrt{-1}$
ω = frequency on radians/sec.
$F(j\omega)$ = Fourier transform, new dependent variable
f_i = voltage of temperature level
i = time increment
τ = column time constant, sec.
$|G(j\omega)|$ = transfer function
$x(t)$ = input function
$y(t)$ = output function
ϕ = phase angle

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